ELSEVIER Linear Algebra and its Applications 312 (2000) 115–123

LINEAR ALGEBRA AND ITS APPLICATIONS

www.elsevier.com/locate/laa

Solving the two-dimensional CIS problem by a rational algorithm☆

Yurii A. Al'pin [a], Alan George [b], Khakim D. Ikramov [c,*]

[a] *Faculty of Mathematics and Mechanics, Kazan State University, 420008 Kazan, Russian Federation*
[b] *Department of Computer Science, University of Waterloo, Waterloo, Ont., Canada*
[c] *Faculty of Computational Mathematics and Cybernetics, Moscow State University, 119899 Moscow, Russian Federation*

Received 3 November 1999; accepted 29 February 2000

Submitted by G.P. Barker

Abstract

The CIS problem is formulated as follows. Let p be a fixed integer, $1 \leqslant p < n$. For given $n \times n$ compex matrices A and B, can one verify whether A and B have a common invariant subspace of dimension p by a procedure employing a finite number of arithmetical operations? We describe an algorithm solving the CIS problem for $p = 2$. Unlike the algorithm proposed earlier by the second and third authors, the new algorithm does not impose any restrictions on A and B. Moreover, when A and B generate a semisimple algebra, the algorithm is able to solve the CIS problem for any p, $1 < p < n$.

Keywords: Common invariant subspace; Rational algorithm; 2-generated matrix algebra; Radical; Socle; Shemesh's theorem

1. Introduction

Let A and B be $n \times n$ complex matrices. The CIS problem (where CIS is an abbreviation for "Common Invariant Subspace") is formulated as follows.

☆ This work was supported by Natural Sciences and Engineering Research Council of Canada grant OGP0008111.

* Corresponding author.
E-mail address: ikramov@cmc.msk.su (K.D. Ikramov).

0024-3795/00/$ - see front matter
PII: S 0 0 2 4 - 3 7 9 5 (0 0) 0 0 0 9 8 - 7